Synthesis and Analysis of Microstrip and Stripline Transmission Line Structures

Project 1

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Abstract

The methodologies for computer calculation of transmission line parameters are studied and formulated into a MATLAB based tool. Transmission line structures of microstrip as stripline are considered. The approach of both synthesis and analysis are formulated for both structure types, and a graphical user interface (GUI) tool is constructed using Matlab.
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Introduction

This report presents an outline for computer calculation of transmission line parameters, considering both microstrip and stripline structures. The methodologies of such calculations are arranged from various literature sources and formulated into a Matlab-based tool. The approach of both synthesis and analysis are formulated for both structure types, and a graphical user interface (GUI) tool is constructed using Matlab.

Synthesis pertains to the formulation of transmission line physical width and length dimensions with an input of various parameters including substrate properties, characteristic impedance, electrical length, and frequency. Analysis considers the calculation of characteristic impedance and line electrical length from input of physical width and length, substrate properties, and frequency. The frequency dependence of effective dielectric constant and characteristic impedance is also presented.

The results of the presented tool are compared to industry standard microwave structure simulators, Ansoft Designer and Ansoft HFSS, for example microstrip and stripline structures. Analysis is compared for all three programs. Synthesis is compared between the provided tool and Designer, as HFSS is strictly an analysis tool. However, the results of the synthesis calculations are analyzed in HFSS as a sort-of check.

Graphical User Interface (GUI)

The GUI accepts numerical inputs from the user as well as a selection for microstrip/stripline and a command for either analysis or synthesis. The numerical inputs can be of a variety of units, selectable from a drop-down window. Substrate parameters of height, relative permittivity (dielectric constant) and loss tangent must be entered, as well as conductor thickness and conductivity. The signal frequency is also a user input.

The results of analysis are characteristic impedance, electrical length, and effective dielectric constant which is sometimes referred to as “Keff.” The results of synthesis are width and physical length.
Equations

The main analytical content of the tool is reported below. The programming code to utilize the equations and techniques, as well as formulate the GUI, is attached with the report.

Stripline

Figure 1 – Graphical User Interface Main View

Figure 2 – Standard Stripline structure representation
Analysis

The characteristic impedance calculation of the stripline transmission line structure is dependent on a ratio of the thickness of the metal to the thickness of the substrate. If the metal thickness is less than $1000^{th}$ of the height, then the structure is considered to be a “thin strip” approximation where the thickness is altogether ignored, and the characteristic impedance is calculated as reported by Cohn in *Characteristic Impedance of the Shielded-Strip Transmission Line*

$$z_0 = \frac{60}{\sqrt{\varepsilon_r}} \cdot \ln\left(\frac{8b}{\pi w}\right)$$

For any thicker signal metallization, a “finite strip” calculation must be followed. This computation was borrowed from a computer program titled *Wcalc* written by Dan McMahill, licensed under the GNU version II, and used with the authors permission.

$$Z_0 = \frac{30}{\sqrt{\varepsilon_r}} \cdot \ln\left(1 + A \cdot \left(2A + \sqrt{4A^2 + 6.27}\right)\right)$$

Where

$$A = \frac{4 \cdot (b - tmet)}{\pi (w - \Delta w)}$$

And

$$\Delta \omega = \frac{tmet}{\pi} \cdot \left(1 - 0.5 \cdot \log\left(\frac{tmet}{2 \cdot b \cdot tmet}\right)^2 + \frac{0.0796 \cdot tmet}{w + 1.1 \cdot tmet}\right)^m$$

and

$$m = 6 \cdot \frac{(b - tmet)}{3b - tmet}$$

The electrical length is computed as a ratio of wavelengths, in degrees, as follows;

$$Electric \ Length = 360 \cdot length \cdot \frac{freq \cdot \sqrt{\varepsilon_r}}{c}$$
The total losses are a combination of the dielectric and conductive losses

\[ \text{loss} = \alpha_d + \alpha_{c, sr} \]

And can be calculated from;

\[ \alpha_d = \alpha_{d1} \times \text{length} \quad (\text{in dB}) \]

Or

\[ \alpha_d = \alpha_{d2} \times \text{length} \quad (Np) \]

Where

\[ \alpha_{d1} = 20 \times \log_{10}(e) \times \alpha_{d1} \]

and

\[ \alpha_{d2} = G \times \frac{z_0}{2} \]

where

\[ G = \frac{2\pi \times \text{tand} \times \text{freq}}{z_0 \times v_p} \]

The conductive losses are much more complicated and utilize an iterative approach, based on the skin depth and wheeler incremental inductance approach, as partly described in Pozar’s *Microwave Engineering* and utilized in the Wcalc code.

\[ \text{skindepth} = \sqrt{\frac{\rho}{\pi \mu_0 f}} \]

If the metal thickness is less than three times the skin depth (tmet < 3*skindepth), Wheeler’s incremental inductance approach is utilized as such;

Find the characteristic impedance \( z_1 \) for the stripline while assuming \( \varepsilon_r = 1 \)

Find the characteristic impedance \( z_2 \) for the stripline while assuming \( \varepsilon_r = 1 \), width = (width - skin depth), and metal thickness = (metal thickness - skin depth)

\[ \alpha_c = \frac{\pi f \times z_2 - z_1}{c \times \frac{z_0}{2}} \]
If \( t_{\text{met}} > 0 \)

\[
R = \alpha_c \times 2 \times z_0
\]

Otherwise, the thickness is approximated as zero and there are no losses

\[
\alpha_c = 0
\]

Converting \( \alpha \) to dB

\[
\alpha_{c,\text{dB}} = 20 \log_{10}(e) \times \alpha_c
\]

Accounting for the surface roughness of the material;

\[
\alpha_{c,\text{sr}} = \alpha_c \left(1 + \frac{2}{\pi} \times \text{atan}(1.4 \times \left(\frac{\text{roughness}}{\text{skindepth}}\right)^2)\right)
\]

**Synthesis**

Synthesis is the analytical computation of transmission line physical width and length given a characteristic impedance, electrical length, and signal frequency from the user input. A process called “bisection steps” for synthesizing the width is borrowed from Wcalc, using the stripline analysis method described above, and computed as follows;

To begin, find the characteristic impedance of the line assuming the width is equal to the height, as used in the analysis function described above in the stripline analysis section. The difference between this impedance and the target impedance is the error value. It is also assumed the maximum value for the width is 100*height and the minimum value of the width is 0.01*height.
\[ error\ val = (computed\ -\ target)\ impedance \]  

Record if the value is positive (greater than the target) or negative (less than the target)

The step size is:

\[ next\ step\ size = \frac{Current\ error\ val - Previous\ error\ val}{Current\ width - Previous\ width} \]

For the initial iteration of synthesis, an assumption is made that the previous width is \(0.99 \times width\), and this new value is used to calculate the “previous” error value. It is computed in this way so that the initial step size is not a value of zero, as this value is in the denominator.

The new width for synthesis is:

\[ width = width - \frac{Error\ val}{step\ size} \]

If this new width is greater than the maximum width \((100 \times height)\) or less than the minimum width \((0.001 \times height)\), take a bisection step;

\[ width = \frac{widthMax + widthMin}{2} \]

The tool analyzes the transmission line (as described in the above) using this new width. The new error value is the computed characteristic impedance minus the user-input or target characteristic impedance. The tool will then check if the new characteristic impedance value is above or below the target value. If the sign of the error value is the same as the initial error value, the analysis has not progressed far enough. If the sign is opposite the initial error value, the analysis has progressed beyond the target value or has “overshot” the analysis.

If the analysis has not progressed far enough, the maximum width is changed to the width used in the previous iteration, since it can be assumed the width needed to produce the target impedance will be less than this value. If the analysis has overshot the value, then the minimum value is changed to the width used in the previous iteration.

This process is repeated to adjust the upper and lower limits of the impedance range. The characteristic impedance error value is set to .00001% of the target.

An example of the refinement of calculated width is captured during runtime of the Matlab tool and plotted below. The calculated width begins very large and decreases until the computation has progressed beyond the target value, in which case the corrective action of the analysis brings the calculated width much closer to the target value in the following iteration.
Finally, the physical length of the stripline is computed similarly to the electrical length as described in the analysis section above. The physical length is computed from the input electrical length and the dielectric constant.

\[
length = \frac{Electric\ Length}{360} \times \frac{c}{freq \times \sqrt{\varepsilon_r}}
\]

**Microstrip**

Unlike the stripline structure presented above, no research or literature reviewed during the development of this tool considered the metallization parameters (thickness and conductivity) during the derivation of closed-form approximations for analysis or synthesis of microstrip transmission line structures. The relationships are too complex to include in simple closed-form approximations and are better handled by field-computation and moment-method-analysis tools designed for the microwave industry. Considering this, the computation of the tool as described below does not consider metallization parameters.
Analysis

The microstrip transmission line structure has metallization placed on top of a substrate and exposed to air, and therefore the “effective” permittivity is a complex relationship between the substrate properties and air properties. As can be seen in the later section titled “Frequency Dependence,” there is also a dependence on signal frequency.

The tool uses the following equation to derive the frequency dependent $K_{eff}$, which were derived by Eikichi Yamashita and available in the paper titled “Tests of Microstrip Dispersion Formulas” by H.A. Atwater.

$$\varepsilon_{re}(f) = \varepsilon_{re}(0) \frac{1 + K f_n^{1.5}}{1 + f_n^{1.5}}$$

Where

$$K = \sqrt{\frac{\varepsilon_r}{\varepsilon_{re}(0)}}$$

and

$$f_n = \left( \frac{4.3 \times f c}{c} \right) \sqrt{\varepsilon_r - 1} \left\{ 0.5 + \left[ 1 + 2 \log_{10} \left( 1 + \frac{w}{h} \right) \right] \right\}$$

This calculation depends on the frequency independent $\varepsilon_{re}$, calculated using equations from the “Transmission Line Structures” notes by Professor Mahmoud El Sabbagh.
\[ \varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + \frac{10h}{\omega} \right)^{-a+b} \]

Where

\[ a = 1 + \frac{1}{49} \ln \left[ \left( \frac{w}{h} \right)^4 + \left( \frac{w}{52 + h} \right)^2 \right] \]
\[ + \frac{1}{18.7} \ln \left[ 1 + \left( \frac{w}{18.1 + h} \right)^3 \right] \]

and

\[ b = 0.564 \left( \frac{\varepsilon_r - 0.9}{\varepsilon_r + 3} \right)^{0.053} \]

The effective epsilon value (\( \varepsilon_{ef} \)) is then used as an input variable to calculate the characteristic impedance value, as shown below. These equations are again from Dr. Sabbagh’s lecture notes. The accuracy of \( Z_0 \) is estimated at better than .03%

\[ Z_0 = \frac{Z_0(\varepsilon_r = 1)}{\sqrt{\varepsilon_{re}}} \]

Where

\[ Z_0(\varepsilon_r = 1) = 60\ln \left[ \frac{F_1 h}{w} + \sqrt{1 + \left( \frac{2h}{w} \right)^2} \right] \]

and

\[ F_1 = 6 + (2\pi - 6)\exp \left\{ -\left( 30.999 \frac{h}{w} \right)^{0.7528} \right\} \]

The electrical length value is calculated from the following equation

\[ \lambda = \frac{c}{f \sqrt{\varepsilon_{re}}} \]

\[ len_{ele} = \frac{len_{phy}}{\lambda} \]
Where $\lambda$ is the effective wavelength of the signal in the microstrip structure, utilizing the $\varepsilon_{ef}$ value. The electrical length is converted to a length represented by degrees using a ratio of the electrical length relative to one wavelength, with one wavelength representing 360 degrees.

**Synthesis**

The synthesis equations utilized by the tool came from *Computer Aided Design* by Gupta in the chapter titled “Characterization of Transmission Structure.” The text provided two equations for Microstrip width based on an arbitrary variable ‘$A$’ being greater than or less than 1.52, with no description or account of where the 1.52 value was derived. Using these two equation “validity regions” allows the tool to minimize numerical error to below two percent.

The value of $A$ is computed from the following equation:

$$A = \frac{Z_0}{60} \left( \frac{\varepsilon_r + 1}{2} \right)^5 + \frac{\varepsilon_r - 1}{\varepsilon_r + 1} (0.23 + \frac{0.11}{\varepsilon_r})$$

When $A > 1.52$ the value for $\frac{w}{h}$ is computed as follows:

$$\frac{w}{h} = \frac{8 \cdot \exp\{A\}}{\exp\{2A\} - 2}$$

When $A \leq 1.52$ the value for $\frac{w}{h}$ is computed as follows:

$$\frac{w}{h} = \frac{2}{\pi} \left\{ B - 1 - l \cdot n + \frac{\varepsilon_r - 1}{2 \varepsilon_r} \left[ l \cdot n (B - 1) + 0.39 - \frac{0.61}{\varepsilon_r} \right] \right\}$$

Where

$$B = \frac{60 \cdot \pi^2}{Z_0 \sqrt{\varepsilon_r}}$$
The physical length value is computed similarly as in analysis, using the following equations:

\[ \lambda = \frac{c}{f \sqrt{\varepsilon_{ef}}} \]

\[ \text{len}_{phy} = \frac{\text{len}_{ele}}{\lambda} \]

**Frequency Dependence**

The frequency dependence of effective dielectric constant and characteristic impedance is also studied for this analysis.

Considering stripline, research has shown that the structure is a TEM structure. Given that the transmission line signal carrying inner conductor is fully encased in the substrate, the “effective” dielectric constant is simply that of the substrate, and the structure is non-dispersive as described on the microwaves101.com stripline page. Therefore there is no dependence of effective dielectric constant on the signal frequency, other than the frequency dependence of the substrate used. Generally a substrate manufacturer will publish these dependencies at a few verified test frequencies. Since the effective dielectric constant is non-frequency-dependent, and the structure is non-dispersive, the characteristic impedance is also non-frequency-dependent.

To graphically represent this analysis, the frequency dependence of characteristic impedance and effective dielectric constant are plotted below. An arbitrary impedance of 50Ω was chosen, and the dielectric constant of Rogers RO 3006 substrate was chosen.
Microstrip transmission line structures have been shown to be not fully TEM and are considered quasi-TEM. The structure will exhibit dispersion across frequency and across any considerable physical length. Research as designed in Atwater has been conducted to develop mathematical closed-form expressions to calculate the frequency dependence of “effective” dielectric constant. The Matlab tool utilizes the conclusions of Eikichi Yamashita et al as shown below.
The characteristic impedance is a function of effective dielectric constant. Our tool utilizes the result of the Yamashita expressions as input for characteristic impedance calculation. Therefore, the Matlab tool considers the frequency dependence of effective dielectric constant as a direct relation to the frequency dependence of characteristic impedance.

The frequency dependence of characteristic impedance and effective dielectric constant are plotted below. The impedance calculation mirrors the included example of comparison of microstrip analysis, and a microstrip structure utilizing Rogers RO 4003 substrate was used.
Comparison to Industry Software

The tool developed here is compared to two industry standard software packages, Ansoft Designer and Ansoft HFSS.

Microstrip Comparison

The input for the microstrip comparison structure contained the following design parameters:

- Height: 400 mils
- Conductivity: 8e9 S/M
- Frequency: 4GHz
- Thickness: 0.7 mils
- Loss Tangent: 0.0027
- Relative Epsilon: 3.55

Analysis

For analysis, the tool received an input width value of 35 mils and a physical length value of 300 mils.
The following is the result of Ansoft Designer analysis:

The following is the result as achieved from HFSS:
The following is the result as achieved from the developed Matlab GUI tool:

The analysis results from all three tools are tabulated in the below table:

<table>
<thead>
<tr>
<th>Tool</th>
<th>Z0 (ohms)</th>
<th>Keff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matlab GUI</td>
<td>57.7</td>
<td>2.75</td>
</tr>
<tr>
<td>Designer</td>
<td>57.08</td>
<td>X</td>
</tr>
<tr>
<td>HFSS</td>
<td>56.3</td>
<td>2.66</td>
</tr>
</tbody>
</table>

Table 1 – Microstrip Analysis results and comparison

The Matlab tool’s characteristic impedance value agrees very closely with Ansoft Designer. HFSS achieved a slightly different result than the Matlab tool and Designer but that is most likely due to the higher accuracy moment of methods approach versus empirical formulas. The Matlab tool’s estimated epsilon effective value is slightly off from the HFSS value but also within a reasonable range.
Synthesis

For synthesis a characteristic impedance value of 50 ohms and an electrical length value of 80 degrees was used as input to the two softwares.

The following is the result of Ansoft Designer analysis:

The following is the result as achieved from HFSS:
The Matlab tool’s physical length value agrees very closely with Ansoft Designer. The width value is subject to a larger amount of error. This may be due to the fact that the equations utilized take frequency dependence into consideration when calculating synthesis values when Ansoft Designer does not take these factors into consideration.

To verify the results of the Matlab tool’s synthesis, a Microstrip structure was designed in HFSS with the same stackup, a width value of 46.96 and a physical length value of 393.87. This structure was analyzed to verify that the characteristic impedance is 50 ohms and the electrical length is 80 degrees. The HFSS results were relatively accurate at 49.3 ohms for the characteristic impedance and the gamma value maps to an accurate electrical length of 82.6. The minor error can be attributed to the fact that HFSS uses the method of moments approach to simulation which takes more factors into consideration than the empirical formulations used in the developed Matlab tool.
**Stripline**

The stripline structure used for comparison had the following design parameters:

- Height: 500 mils
- Frequency: 3GHz
- Conductivity: 8e9 S/M
- Thickness: 0.7 mils
- Loss Tangent: 0.0025
- Relative Epsilon: 6.15

**Analysis**

For analysis the tool received a width value 20 mils and a physical length value of 300 mils.

The following is the result of Ansoft Designer analysis:
The following is the result as achieved from HFSS:

The following is the result as achieved from the developed Matlab GUI tool:
The analysis results from all three tools are tabulated in the below table:

<table>
<thead>
<tr>
<th>Tool</th>
<th>Z0 (ohms)</th>
<th>Keff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matlab GUI</td>
<td>43.75</td>
<td>6.15</td>
</tr>
<tr>
<td>Designer</td>
<td>43.75</td>
<td>X</td>
</tr>
<tr>
<td>HFSS</td>
<td>49.31</td>
<td>6.16</td>
</tr>
</tbody>
</table>

Table 3 – Stripline Analysis results and comparison

The Matlab tool’s characteristic impedance value agrees extremely closely with Ansoft Designer. HFSS achieved a slightly different result than the Matlab tool and Designer but that is most likely due to the higher accuracy moment of methods approach versus empirical formulas. The Matlab tool’s estimated epsilon effective value is also very accurate compared to HFSS.

**Synthesis**

For synthesis, a characteristic impedance value of 50 ohms and an electrical length value of 80 degrees was used as input to the two softwares.

The following is the result of Ansoft Designer analysis:
The following is the result as achieved from HFSS:

The following is the result as achieved from the Matlab GUI tool:

The analysis results from all three tools are tabulated in the below table:
<table>
<thead>
<tr>
<th>Tool</th>
<th>Width (mils)</th>
<th>Physical Length (mils)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matlab GUI</td>
<td>9.33</td>
<td>616.95</td>
</tr>
<tr>
<td>Designer</td>
<td>9.33</td>
<td>616.95</td>
</tr>
</tbody>
</table>

Table 4 – Stripline Synthesis results and comparison

The Matlab tool’s results are extremely accurate for width and physical length compared to the results achieved from Ansoft Designer. Any minor errors are easily lost in the noise of the synthesis calculation.

To verify the results of the synthesis in HFSS, a Stripline structure was designed in HFSS with the same stackup, a width value of 9.33 and a physical length value of 616.95. This structure was analyzed to verify that the characteristic impedance is 60 ohms and the electrical length is 140 degrees. The HFSS results were relatively similar at 61.75 ohms for the characteristic impedance and the gamma value relates to an accurate electrical length value of 140.28 degrees. The minor error can be attributed to the fact that HFSS uses the method of moments approach to simulation which takes more factors into consideration than the utilized empirical formulations.

**Conclusion**

The analysis and synthesis results achieved from the developed Matlab GUI application agree with simulation results obtained from Ansoft designer and HFSS within a reasonable degree of error. There is a high degree of confident in the accuracy of the utilized equations for Microstrip and Stripline circuits.

As the plots of characteristic impedance and estimated epsilon effective versus frequency provided expected results for both Microstrip and Stripline cases. The characteristic impedance of Microstrip and Stripline circuits had a direct relationship with frequency. The estimated effective epsilon value increased as a low order exponential relationship with respect to frequency. The estimated effective epsilon value of Stripline was constant with respect to frequency which is to be expected.
**Bibliography**


